THE LUCKY PRIME NUMBER THEOREM

Ing. Pier Franz Roggero, Dott. Michele Nardelli, P.A. Francesco Di Noto

Abstract:

In this paper is described the asymptotic distribution of the lucky prime number with the lucky prime number theorem.
Index:

1. THE LUCKY PRIME NUMBER THEOREM ................................................................. 3
2. REFERENCES ........................................................................................................ 5
1. THE LUCKY PRIME NUMBER THEOREM

A **lucky number** is a natural number in a set which is generated by a certain "sieve". This sieve is similar to the **Sieve of Eratosthenes** that generates the prime numbers, but it eliminates numbers based on their position in the remaining set, instead of their value (or position in the initial set of natural numbers).

A "**lucky prime**" is a lucky number which is also prime.

Let \( \pi(n) \) be the prime-counting lucky function that gives the number of lucky primes less than or equal to \( n \):

\[
\Pi(n) \approx \frac{3n}{2 \ln^2(n)}
\]

So we have that the \( n \) th lucky prime number \( L_n \) satisfies:

\[
L_n \approx \frac{2}{3} n \ln^2(n)
\]

The \( n \)-th lucky prime gap, denoted \( \Delta_n \) is the difference between the \((n + 1)\)-th and the \( n \)-th lucky prime numbers:

\[
\Delta_n < \frac{4}{9} \ln^4(n)
\]

The values are lower to the maximum lucky prime gap \( \Delta_{MAXn} = \frac{4}{9} \ln^4(n) \).

**Of course with this theorem there are infinitely many lucky primes.**

In tab. 1 we have a list of \( n \) from 10 to 1,000,000,000:
<table>
<thead>
<tr>
<th>n</th>
<th>COUNTING PRIME LUCKY NUMBER</th>
<th>COUNTING LUCKY PRIME NUMBER ESTIMATED (3/2*n/ln(n)^2)</th>
<th>n th LUCKY PRIME NUMBER</th>
<th>n th LUCKY PRIME NUMBER ESTIMATED 2/3<em>n</em>ln(n)^2</th>
<th>MAX GAP LUCKY PRIME NUMBER</th>
<th>MAX GAP LUCKY PRIME NUMBER ESTIMATED 4/9*ln(n)^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2</td>
<td>2,8</td>
<td>127</td>
<td>35,3</td>
<td>48</td>
<td>12,5</td>
</tr>
<tr>
<td>100</td>
<td>9</td>
<td>7,1</td>
<td>3307</td>
<td>1413,8</td>
<td>156</td>
<td>199,9</td>
</tr>
<tr>
<td>1000</td>
<td>43</td>
<td>31,4</td>
<td>75193</td>
<td>31811,4</td>
<td>654</td>
<td>1012,0</td>
</tr>
<tr>
<td>10000</td>
<td>211</td>
<td>176,8</td>
<td>1191163</td>
<td>565535,8</td>
<td>1344</td>
<td>3198,3</td>
</tr>
<tr>
<td>100000</td>
<td>1300</td>
<td>1131,7</td>
<td>8836496,9</td>
<td>7808,4</td>
<td>3198,3</td>
<td>7808,4</td>
</tr>
<tr>
<td>1000000</td>
<td>8616</td>
<td>7858,8</td>
<td>127245554,7</td>
<td>16191,4</td>
<td>29996,6</td>
<td>16191,4</td>
</tr>
<tr>
<td>1E+07</td>
<td>62446</td>
<td>57738,3</td>
<td>1731953382,8</td>
<td>29996,6</td>
<td>29996,6</td>
<td>29996,6</td>
</tr>
<tr>
<td>1E+08</td>
<td>469146</td>
<td>442058,7</td>
<td>22621431938,0</td>
<td>51172,9</td>
<td>51172,9</td>
<td>51172,9</td>
</tr>
<tr>
<td>1E+09</td>
<td>3656784</td>
<td>3492809,2</td>
<td>286302497965,8</td>
<td>81969,1</td>
<td>81969,1</td>
<td>81969,1</td>
</tr>
</tbody>
</table>
2. REFERENCES

1) Weisstein, Eric W. "Lucky Number". MathWorld.